Unit 6 Lesson 2A Sum + Difference Formulas

I. Warm-Up

$$\boxed{\begin{aligned}
\cos(\alpha + B) &= \cos\alpha \cos B - \sin\alpha \sin B \\
\cos(\alpha - B) &= \cos\alpha \cos B + \sin\alpha \sin B \\
\\
\sin(\alpha + B) &= \sin\alpha \cos B + \cos\alpha \sin B \\
\sin(\alpha - B) &= \sin\alpha \cos B - \cos\alpha \sin B
\end{aligned}}$$

What patterns do you notice?

How can these help you commit them to memory?

II. Objective:

Today you will explore where these formulas derive from and how to apply them to evaluate angles.

Homework: Your homework is to study for your quiz tomorrow on Unit 6 Lesson 1. Use the practice given on Thursday.

Help: There are videos on the wikispace that walk through some problems. Mrs. Pike can also be reached by email.

II. Video: Cosine of the Difference of Two Angles

(Why? It helps to know we didn't just make this up.)

* During the video write down three things that strike your interest

  .

  .

  .

Try the proof on your own.

Manipulate, You can use prop. of equality b/c we know this is true

$$\boxed{2-2\cdot\cos(\alpha-B) = (\cos\alpha-\cos B)^2 + (\sin\alpha-\sin B)^2}$$

$$\boxed{\cos(\alpha-B) = \cos\alpha\cos B+\sin\alpha\sin B}$$

IV. Using Dif. of Cosine to derive others.

With you partner take 5 minutes to justify each step.

A. $\cos(\alpha + B) = \cos\alpha \cos B - \sin\alpha \sin B$ Given

$= \cos(\alpha - (-B))$ ____________

$= \cos\alpha \cos(-B) + \sin\alpha \sin(-B)$ ____________

$= \cos\alpha \cos(B) + \sin\alpha \sin(-B)$ ____________

$= \cos\alpha \cos B - \sin\alpha \sin B$ ____________

B. $\sin(\alpha + B) = \sin\alpha \cos B + \cos\alpha \sin B$ Given

$= \cos\left(\frac{\pi}{2} - (\alpha + B)\right)$ Cofunction Id. Why is $\sin x = \cos\left(\frac{\pi}{2} - x\right)$?

$= \cos\left(\frac{\pi}{2} - \alpha - B\right)$ ____________

$= \cos\left(\left(\frac{\pi}{2} - \alpha\right) - B\right)$ ____________

$= \cos\left(\frac{\pi}{2} - \alpha\right) \cos B + \sin\left(\frac{\pi}{2} - \alpha\right) \sin B$ ____________

$= \sin\alpha \cos B + \cos\alpha \sin B$ ____________.

Turn & Talk with another group to compare.

I. Practice

You should complete this individually or with a partner. If time allows, please volunteer to put your work for a problem on the board.

Practice Exercises

Use the formula for the cosine of the difference of two angles to solve Exercises 1–12.

In Exercises 1–4, find the exact value of each expression.

1. $\cos(45° - 30°)$
2. $\cos(120° - 45°)$
3. $\cos\left(\frac{3\pi}{4} - \frac{\pi}{6}\right)$
4. $\cos\left(\frac{2\pi}{3} - \frac{\pi}{6}\right)$

In Exercises 5–8, each expression is the right side of the formula for $\cos(\alpha - \beta)$ with particular values for α and β.

a. *Identify α and β in each expression.*

b. *Write the expression as the cosine of an angle.*

c. *Find the exact value of the expression.*

5. $\cos 50° \cos 20° + \sin 50° \sin 20°$
6. $\cos 50° \cos 5° + \sin 50° \sin 5°$
7. $\cos\frac{5\pi}{12}\cos\frac{\pi}{12} + \sin\frac{5\pi}{12}\sin\frac{\pi}{12}$
8. $\cos\frac{5\pi}{18}\cos\frac{\pi}{9} + \sin\frac{5\pi}{18}\sin\frac{\pi}{9}$

In Exercises 9–12, verify each identity.

9. $\frac{\cos(\alpha - \beta)}{\cos\alpha\sin\beta} = \tan\alpha + \cot\beta$
10. $\frac{\cos(\alpha - \beta)}{\sin\alpha\sin\beta} = \cot\alpha\cot\beta + 1$
11. $\cos\left(x - \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}(\cos x + \sin x)$
12. $\cos\left(x - \frac{5\pi}{4}\right) = -\frac{\sqrt{2}}{2}(\cos x + \sin x)$

You do not need a calculator. Use your unit circle knowledge!

Time to Spare? Practice for quiz tomorrow.